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# PHASE TRANSITIONS OF A QUASI-ONE-DIMENSIONAL FERMI GAS

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Phase transitions of a quasi-one-dimensional Fermi gas are studied by the use of the renormalization group method for the fermion system without the bosonization. Intrachain backward and forward scatterings, pair electron hopping between different chains and exchange scattering of electrons in different chains are taken account of. Pair electron hopping gives rise to the singlet and triplet superconductor phase transitions. Electron exchange scattering gives rise to the spin density wave and charge density wave transitions. Calculation is made by the use of the one-dimensional dispersion. Following Menyhárd's method (Ref. 1) for the model with interchain backward scattering only, the change of the fluctuation from the one-dimensional region to the three-dimensional region is described. The transition temperatures are given by the bare couplings at high temperatures.

## INTRODUCTION

In this paper, we study phase transitions of a quasi-one-dimensional (Q-1-D) Fermi gas with one-dimensional (1-D) dispersion by the use of the renormalization group (RG) without the bosonization. The model we

consider has intrachain backward and forward scatterings, pair electron hopping between different chains and exchange scattering of electrons in different chains.

We think the Q-1-D Fermi gas system as follows. In high temperatures, the intrachain coupling is much stronger than the interchain coupling, i.e. $\gamma_{i\neq j}/\gamma_{ii} \ll 1$ $(i \neq j)$ ($\gamma_{i\neq j}$ and $\gamma_{ii}$ are interchain and intrachain bare couplings respectively). As temperature decreases, interchain coupling becomes stronger and comparable to the intrachain coupling at a crossover temperature. Below the crossover temperature, the correlation is three-dimensional (3-D). We describe the above steps by using Menyhárd's method[1] for the system with interchain backward scattering only. For the 3-D region, we treat the intrachain and interchain couplings on an equal footing by the use of the first order RG method.

We neglect single electron hopping.

*The results obtained by this paper are as follows.* Pair electron hopping gives rise to the singlet and triplet superconductor (SS and TS) phase transitions. Electron exchange scattering gives rise to the spin density wave (SDW) and charge density wave (CDW) phase transitions. The crossover temperatures in which the interchain couplings become comparable to the intrachain couplings and the transition temperatures are given by the bare couplings at high temperatures.

## MODEL AND RG

The model we investigate is given by the following Hamiltonian.

$$H = H_0 + H_1 + H_2 , \tag{1}$$

$$H_0 = \sum_{k,\alpha,i} [\, v_F(k-k_F) a^+_{ik\alpha} a_{ik\alpha} + v_F(-k-k_F) b^+_{ik\alpha} b_{ik\alpha} \,], \tag{2}$$

$$H_1 = \frac{1}{L} \sum_{i} \sum_{k_1,k_2,q} \sum_{s s'} (g_{1i} a^+_{i,k_1+q,s} b^+_{i,k_2-q,s'} a_{ik_2 s'} b_{ik_1 s} - g_{2i} a^+_{ik_1+q,s} b^+_{i,k_2-q,s'} a_{ik_1 s} b_{ik_2 s'}) \tag{3}$$

$$H_2=\frac{1}{L}\sum_{i,j}\sum_{k_1,k_2,q}\sum_{ss'}(K_{1ij}a^{+}_{i,k_1+q,s}b^{+}_{i,k_2-q,s'}a_{jk_2s'}b_{jk_1s}$$
$$-K_{2ij}a^{+}_{i,k_1+q,s}b^{+}_{i,k_2-q,s'}a_{jk_1s}b_{jk_2s'}$$
$$+L_{1ij}a^{+}_{i,k_1+q,s}b^{+}_{j,k_2-q,s'}a_{ik_2s'}b_{jk_1s}$$
$$-L_{2ij}a^{+}_{i,k_1+q,s}b^{+}_{j,k_2-q,s'}a_{jk_2s}b_{ik_1s'}), \quad (4)$$

where $a_k$ and $b_k$ are annihilation operators for right-and left-going electrons. Further, i and j are chain indices and s and s' spin indices. Here we do not study the interchain backward and forward scatterings. An effect of backward scattering was studied by Mihály and Sólyom[2] and Menyhárd.[1] An effect of forward scattering was investigated by Lee et al.[3] The Fermi surface we use is flat. The Fermi surface is not distorted since we have neglected single electron hopping.

The RG equations for invariant couplings are

$$\frac{\partial g^{*}_{1i}}{\partial \ell}=2g^{*2}_{1i}+2g^{*3}_{1i}+2\sum_{k\neq i}K^{*}_{1ik}K^{*}_{2ki}, \quad (5)$$

$$\frac{\partial g^{*}_{2i}}{\partial \ell}=g^{*2}_{1i}+g^{*3}_{1i}+\sum_{k\neq i}(K^{*2}_{1ik}+K^{*2}_{2ik}-L^{*2}_{2ik}), \quad (6)$$

$$\frac{\partial K^{*}_{1ij}}{\partial \ell}=4L^{*}_{1ij}K^{*}_{1ij}+K^{*}_{1ij}(g^{*}_{2i}+g^{*}_{2j}+F)+K^{*}_{2ij}(g^{*}_{1i}+g^{*}_{1j})$$
$$-2(K^{*}_{1ij}J^{*}_{2ij}+L^{*}_{1ij}K^{*}_{2ij})+\sum_{k\neq i,j}(K^{*}_{1ik}K^{*}_{2kj}+K^{*}_{2ik}K^{*}_{1kj}), \quad (7)$$

$$\frac{\partial K^{*}_{2ij}}{\partial \ell}=K^{*}_{1ij}(g^{*}_{1i}+g^{*}_{1j})+K^{*}_{2ij}(g^{*}_{2i}+g^{*}_{2j}+F)-2J^{*}_{2ij}K^{*}_{2ij}$$
$$+\sum_{k\neq i,j}(K^{*}_{1ik}K^{*}_{1kj}+K^{*}_{2ik}K^{*}_{2kj}), \quad (8)$$

$$\frac{\partial L^{*}_{2ij}}{\partial \ell}=-L^{*}_{2ij}(g^{*}_{2i}+g^{*}_{2j}+F)-\sum_{k\neq i,j}L^{*}_{2ik}L^{*}_{2kj}, \quad (9)$$

$$F=2(g^{*2}_{1i}+g^{*2}_{2i}-g^{*}_{1i}g^{*}_{2i}) \quad (10)$$

$2\pi v_F=1$, $\ell=\ln(T/D)$, $v_F$:Fermi velocity, D: cutoff,

where we have calculated the equations up to the lowest order with respect to the interchain couplings and up to the next order with respect to the intrachain couplings.

Further we have neglected the coupling $L_{1ij}$ by considering that $L_{1ij}$ does not contribute to divergences of correlation functions (see the following equations (11)~(14)). Here $A^*$ denotes the invariant coupling that corresponds to a bare coupling constant A.

Next we consider the four correlation functions of order parameters: $\Delta_S$ is one for SS, $\Delta_T$ TS, $\chi$ SDW and N CDW. We obtain the following Lie equations for correlation functions.

$$\frac{\partial}{\partial \ell}\ln\overline{\Delta}_{Sij}=2K^*_{1ij}+2K^*_{2ij}+(2g^*_{1i}+2g^*_{2i}+F)\delta_{ij}\ , \tag{11}$$

$$\frac{\partial}{\partial \ell}\ln\overline{\Delta}_{Tij}=-2K^*_{1ij}+2K^*_{2ij}+(-2g^*_{1i}+2g^*_{2i}+F)\delta_{ij}\ , \tag{12}$$

$$\frac{\partial}{\partial \ell}\ln\overline{\chi}_{ij}=-2L^*_{2ij}-(2g^*_{2i}+F)\delta_{ij}\ , \tag{13}$$

$$\frac{\partial}{\partial \ell}\ln\overline{N}_{ij}=-2L^*_{2ij}+(4g^*_{1i}-2g^*_{2i}+F)\delta_{ij} \tag{14}$$

$$\overline{R}_{ij}=\pi v_F\frac{\partial R_{\lambda ij}}{\partial \ell}$$

where $R_{ij}$ is any of correlation functions.

## 1-D AND 3-D CORRELATIONS AND TRANSITION TEMPERATURES

Following Menyhárd, we study the effect of the 1-D correlation on the phase transition of the Q-1-D Fermi gas. We make the assumption, $\gamma_{\lambda ij} \ll \gamma_{\lambda ii} \ll 1$ $(i\neq j)$ $(2\pi v_F=1)$, in the bare couplings. In the region $T \gtrsim T^{cro}$ ($T^{cro}$: a crossover temperature), we assume that the values of the interchain couplings are smaller than those of the intrachain couplings. We treat the intrachain couplings of the system with $g_1<0$ by the second order RG and the interchain couplings by the first order RG. But we treat the intrachain couplings of the system with $g_1>0$ by the first order RG, since the intrachain couplings remain weak in the whole temperature region when $g_1>0$. We assume that the intrachain couplings are not renormalized by the interchain couplings in the region $T \gtrsim T^{cro}$.

By the use of this assumption, we obtain the mean field transition temperatures from the divergences of the renormalized interchain couplings as

$$T_{SS}^{MF} = T_{1D}\left[\frac{2A}{\gamma}|K^0|\right]^{1/\gamma} \tag{15}$$

$$T_{SDW}^{MF} = T_{1D}\left[\frac{2A}{\gamma}L^0\right]^{1/\gamma} \tag{16}$$

where the characteristic temperature $T_{1D}$ is given by

$$T_{1D} = D|g_1|^{1/2}\exp[-1/2|g_1|] \tag{17}$$

for $g_1<0$ ($g_1=g_{1i}$, $i=1,\cdots,N$) and $T_{1D}=D$ for $g_1>0$ and

$$A=|g_1|^{-3/2},\ \gamma=3/2, \quad \text{for } g_1<0$$

$$A=1,\ \gamma=|g_1-2g_2|, \quad \text{for } g_1>0 \tag{18}$$

Further, in Eqs. (15) and (16), $|\gamma^0|=\max|\gamma(0,q)|$ ($\gamma$=K or L), $K^0<0$ and $L^0>0$ where $\gamma(0,q)$ is the Fourier transform of bare coupling $\gamma_{\lambda\lambda'}(0)$ ($K_{\lambda\lambda'}=K_{1\lambda\lambda'}=K_{2\lambda\lambda'}$, $L_{\lambda\lambda'}=L_{\lambda\lambda\lambda'\lambda'}$).

On the other hand, we assume that the interchain couplings are comparable to the intrachain couplings in the region $T_c<T<T^{cro}$ ($T_c$: a critical temperature). In this region, the intrachain couplings are renormalized by the interchain couplings.

Let us define the crossover temperatures $T_{SS}^{cro}$, $T_{SDW}^{cro}$ and $T_{CDW}^{cro}$ by temperatures in which the coutributions of the interchain couplings become equal to ones of the intrachain couplings in Eqs. (11), (13) and (14). We obtain

$$T_{SS}^{cro} = T_{SS}^{MF}\left[\gamma/|g_1^*(T_{SS}^{cro})+g_2^*(T_{SS}^{cro})|+1\right]^{1/\gamma} \tag{19}$$

$$T_{SDW}^{cro} = T_{SDW}^{MF}\left[\gamma/2g_2^*(T_{SDW}^{cro})+1\right]^{1/\gamma} \tag{20}$$

$$T_{CDW}^{cro} = T_{SDW}^{MF}\left[\frac{1}{2}\gamma/|2g_1^*(T_{CDW}^{cro})-g_2^*(T_{CDW}^{cro})|+1\right]^{1/\gamma} \tag{21}$$

where $g_1^*(T_{SS}^{cro})+g_2^*(T_{SS}^{cro})<0$, $g_2^*(T_{SDW}^{cro})>0$ or $2g_1^*(T_{CDW}^{cro})-g_2^*(T_{CDW}^{cro})<0$. When $g_1^*(T_{SS}^{cro})+g_2^*(T_{SS}^{cro})>0$, $g_2^*(T_{SDW}^{cro})>0$

or $2g_1^*(T_{CDW}^{cro})-g_2^*(T_{CDW}^{cro})>0$, the crossover temperature, $T_{SS}^{cro}$, $T_{SDW}^{cro}$ or $T_{CDW}^{cro}$, is not given. The intrachain couplings $g_1^*(T^{cro})$ and $g_2^*(T^{cro})$ are calculated by using the second order RG of the 1-D system and given by the intrachain and interchain bare couplings. The crossover temperature is also given by the bare couplings.

We consider the temperature region $T< T^{cro}$ and treat the intrachain and interchain couplings on an equal footing. We take couplings as

$$g_{1i}=\gamma_{ij}^{K}\delta_{ij}\ ,\quad g_{2i}=(\gamma_{ij}^{K}+\gamma_{ij}^{L})\delta_{ij}\ , \tag{22}$$

$$K_{ij}=\gamma_{ij}^{K}\ ,\quad L_{ij}=\gamma_{ij}^{L}\quad (i\neq j)\ . \tag{23}$$

We use the couplings at the crossover temperatures as new bare couplings, i.e. $\gamma^K(0,q)=g_1^*(T_{SS}^{cro})+g_2^*(T_{SS}^{cro})<0$, $\frac{1}{2}\gamma_{SDW}^{L}(0,q)=g_2^*(T_{SDW}^{cro})>0$ and $\frac{1}{2}\gamma_{CDW}^{L}(0,q)=|2g_1^*(T_{CDW}^{cro})-g_2^*(T_{CDW}^{cro})|$ where $\gamma(0,q)$ is the Fourier transform of $\gamma_{ij}$. Further, $\ell$ is replaced by $\ell=\ln(T/T^{cro})$. Then we obtain the transition temperatures of SS, SDW and CDW from the temperatures, where $\gamma^{K*}(\ell,q)$, $\gamma_{SDW}^{L*}(\ell,q)$ and $\gamma_{CDW}^{L*}(\ell,q)$ diverge, as

$$T_s=T_{SS}^{cro}\exp[-1/2|\gamma^{K}(0,q)|]\ , \tag{24}$$

$$T_{p_1}=T_{SDW}^{cro}\exp[-1/\gamma_{SDW}^{L}(0,q)]\ , \tag{25}$$

$$T_{p_2}=T_{CDW}^{cro}\exp[-1/\gamma_{CDW}^{L}(0,q)]\ , \tag{26}$$

In Eqs. (24)$\sim$(26), the transition temperatures have q-dependences. When $\gamma^K(0,q)$ has a minimum value at $q=Q_1$, the highest $T_s$ is attained at $q=Q_1$. When $\gamma_{SDW}^{L}(0,q)$ or $\gamma_{CDW}^{L}(0,q)$ has a maximum value at $q=Q_2$, the highest $T_{p_1}$ or $T_{p_2}$ is attained at $q=Q_2$. Also, $q=0$ or $\frac{\pi}{a}$ (a: the distance between the nearest neighbor chains) respectively shows that the ordered states of the nearest neighbor chains are in the same or opposite phases.

Next we consider the correlation functions. By the use of the transform of correlation functions, we obtain

$$\Delta_s(\ell,q)\quad[1-2\ell\gamma^{K}(0,q)]^{-1}, \tag{27}$$

$$\chi(\ell, q) \quad [1+\ell\, \gamma^{L}_{SDW}(0, q)]^{-1}, \tag{28}$$

$$N(\ell, q) \quad [1+\ell\, \gamma^{L}_{CDW}(0, q)]^{-1}, \tag{29}$$

where $\Delta(\ell, q)$, $\chi(\ell, q)$ and $N(\ell, q)$ diverge at $T=T_S$, $T=T_{P_1}$ and $T=T_{P_2}$ respectively. That is, electron exchange scattering gives rise to the SDW and CDW phase transitions. Here we notice that the SDW phase transition is given rise to by the exchange of electrons with different spins, while the CDW phase transition by the exchange of electrons with same spins which is similar to the Coulomb type scattering between electrons in different chains.

The TS phase transition does not come into existence when $K_{1\lambda j} = K_{2\lambda j}$. For example, when one takes $K_{\lambda j} = -K_{1\lambda j} = K_{2\lambda j}$, one obtains the same result as Eq. (27) for $\Delta_T$ instead of $\Delta_S$.

## CONCLUDING REMARKS

Various kinds of phase transitions come about in the following bare coupling regions. The SS (TS) phase transition at finite temperature comes about in the coupling region, $[g_1 - 2g_2 > 0$ or $g_1 < 0]$ and $K^{\rho} < 0$ where $K_{\lambda j} = K_{1\lambda j} = K_{2\lambda j}$ ($K_{\lambda j} = -K_{1\lambda j} = K_{2\lambda j}$), the SDW transition in the region, $g_1 > 0$, $g_1 - 2g_2 < 0$ and $L^{\rho} > 0$, and the CDW transition in the region, $[g_1 - 2g_2 < 0$ or $g_1 < 0]$ and $L^{\rho} > 0$.

The 3-D temperature regions are wide in the system with $g_1 > 0$, since the critical temperatures are given by Eqs. (24)~(26) and the new bare couplings are $|\gamma^{K}(0, q)| \ll 1$, $|\gamma^{L}_{SDW}(0, q)| \ll 1$ and $|\gamma^{L}_{CDW}(0, q)| \ll 1$ for the system with $g_1 > 0$. Therefore it especially need for the system with $g_1 > 0$ in which one treats the intrachain and interchain couplings on an equal footing to obtain the critical temperatures. On the other hand, the 3-D regions are narrow in the system $g_1 < 0$ since $|\gamma^{K}(0, q)| \sim 0(1)$, $|\gamma^{L}_{SDW}(0, q)| \sim 0(1)$ and $|\gamma^{L}_{CDW}(0, q)| \sim 0(1)$.

Finally, we state the following. The method[5)~8)] where one takes account of the intrachain coupling exactly and treats the interchain coupling in the mean

field approximation describes the region $T>T^{CTO}$, but does not describe the region $T_c < T < T^{CTO}$.

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